Please check the examination details below before entering your candidate information

Candidate surname   Other names

Pearson Edexcel
International GCSE

Centre Number

Candidate Number

Wednesday 15 January 2020

Morning (Time: 2 hours)　　　　　Paper Reference **4MA1/2HR**

Mathematics A
Paper 2HR
Higher Tier

You must have:
Ruler graduated in centimetres and millimetres, protractor, compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- **Calculators may be used.**
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

P 5 9 8 1 7 A 0 1 2 8

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International GCSE Mathematics

### Formulae sheet – Higher Tier

Arithmetic series

Sum to n terms, $S_n = \dfrac{n}{2} \; [2a + (n - 1)d]$

The quadratic equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Area of trapezium $= \dfrac{1}{2}(a + b)h$
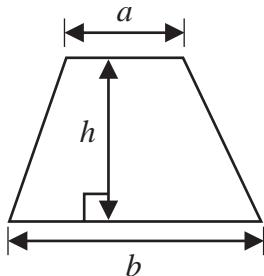

Trigonometry
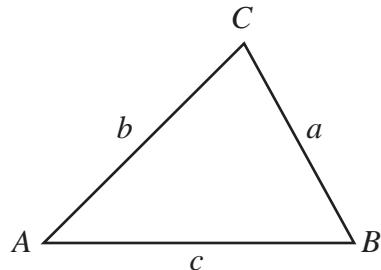
In any triangle ABC

Sine Rule $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine Rule $a^2 = b^2 + c^2 - 2bc \cos A$

**Area of triangle** $= \dfrac{1}{2} ab \sin C$

Volume of cone $= \dfrac{1}{3} \pi r^2 h$

**Curved surface area of cone** $= \pi r l$
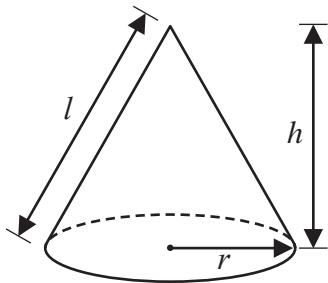
Volume of prism
= area of cross section × length
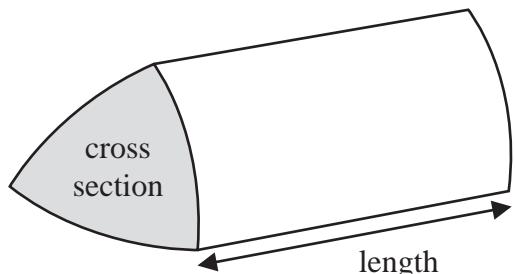
Volume of cylinder $= \pi r^2 h$
**Curved surface area
of cylinder** $= 2\pi r h$
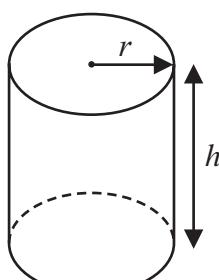
Volume of sphere $= \dfrac{4}{3} \pi r^3$

Surface area of sphere $= 4\pi r^2$
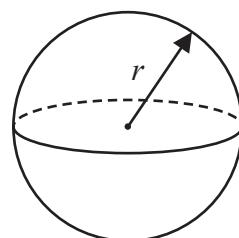
Answer ALL TWENTY SIX questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 (a) Write $5^{17} \times 5^2$ as a single power of 5

$5^{17+2} = 5^{19}$

5^{19} ①
....

(1)

(b) Write 800 as a product of its prime factors.
Show your working clearly.

800

2   400

2   200

2   100   ①

2   50

2   25

5   5

$2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 = 800$

$2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5$ ①
....

(2)
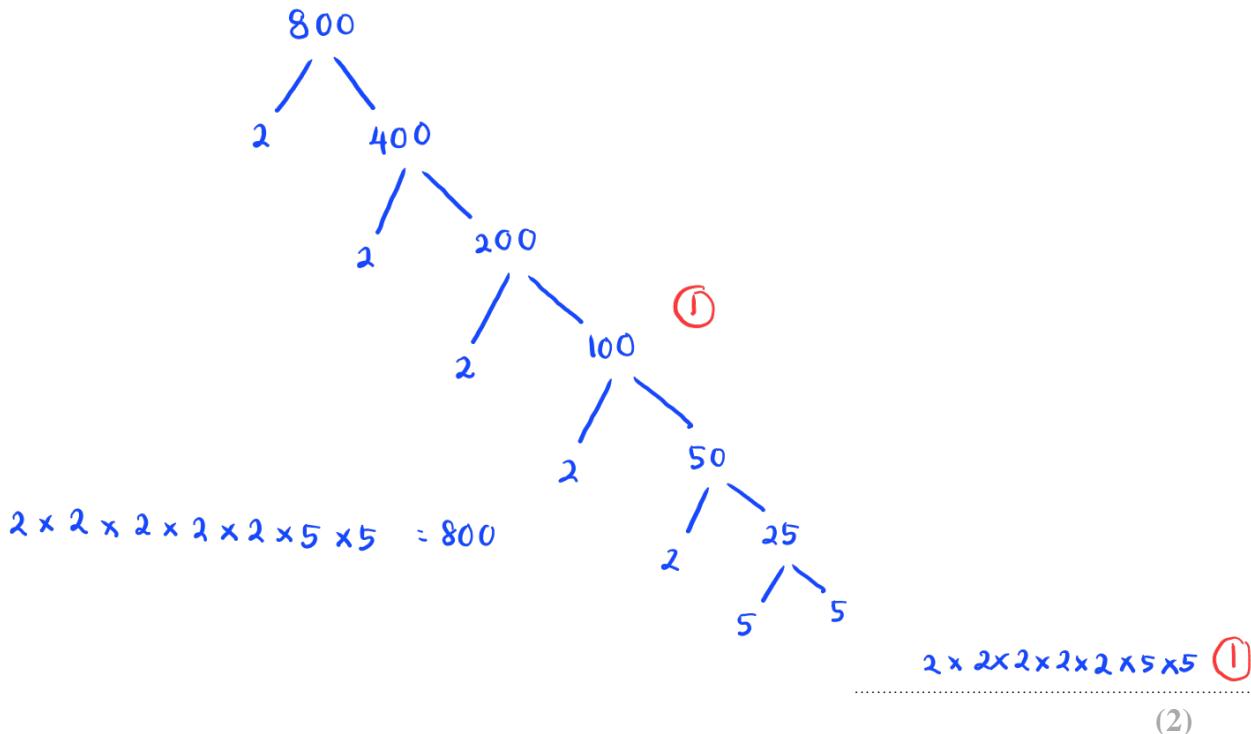
(Total for Question 1 is 3 marks)

2 The table gives information about the amount of money, in £, that Fiona spent in a grocery store each week during 2019

Amount spent (£x)	Frequency
$0 \leqslant x < 20$	5
$20 \leqslant x < 40$	11
$40 \leqslant x < 60$	8
$60 \leqslant x < 80$	19
$80 \leqslant x < 100$	9

Work out an estimate for the total amount of money that Fiona spent in the grocery store during 2019

Total estimation : $\sum$ midpoint × frequency for all classes

Total $= (10 \times 5) + (30 \times 11) + (50 \times 8) + (70 \times 19) + (90 \times 9)$ ①

$= 50 + 330 + 400 + 1330 + 810$ ①

$= 2920$ ①

£ 2920

(Total for Question 2 is 3 marks)

P 5 9 8 1 7 A 0 4 2 8

3 Three tins, *A*, *B* and *C*, each contain buttons.

Tin *A* contains *x* buttons.
Tin *B* contains 4 times the number of buttons that tin *A* contains.
Tin *C* contains 7 fewer buttons than tin *A*.

The total number of buttons in the three tins is 137

Work out the number of buttons in tin *C*.

$A = x$

$B = 4x$ ①

$C = x - 7$

$\text{Total} = A + B + C$

$= x + 4x + (x-7) = 137$ ①

$\therefore 6x = 137 + 7$

$6x = 144$

$x = \dfrac{144}{6} = 24$ ①

$C = 24 - 7$

$= 17$ ①

17

(Total for Question 3 is 4 marks)

**4** The diagram shows a rectangle and a diagonal of the rectangle.

8.5 cm

Diagram **NOT** accurately drawn

5.6 cm
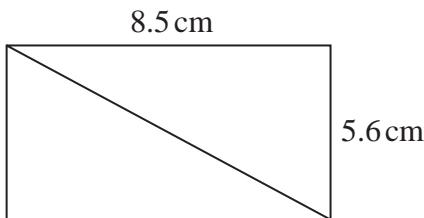
Work out the length of the diagonal of the rectangle.
Give your answer correct to 1 decimal place.

Using Pythagoras' theorem :

$$\text{diagonal} = \sqrt{8.5^2 + 5.6^2} \quad ①$$
$$= \sqrt{103.61} \quad ①$$
$$= 10.2 \quad ①$$

10.2 .... cm

(Total for Question 4 is 3 marks)

5 A plane takes 3 hours 36 minutes to fly from the Cayman Islands to New York.
The plane flies a distance of 2470 km.

Work out the average speed of the plane in km/h.
Give your answer correct to the nearest whole number.

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

$$36 \text{ minutes} \times \frac{1 \text{ hour}}{60 \text{ minutes}} = 0.6 \text{ hour} \quad ①$$

×60
hour → minutes
÷60
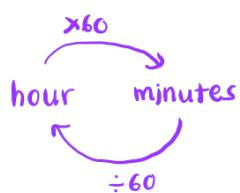
$$\text{Speed} = \frac{2470 \text{ km}}{3.6 \text{ h}} \quad ①$$
$$= 686 \text{ km/h} \quad ①$$

686 .... km/h

(Total for Question 5 is 3 marks)

6 Use ruler and compasses only to construct the perpendicular bisector of the line *AB*.
You must show all your construction lines.

A

B

②
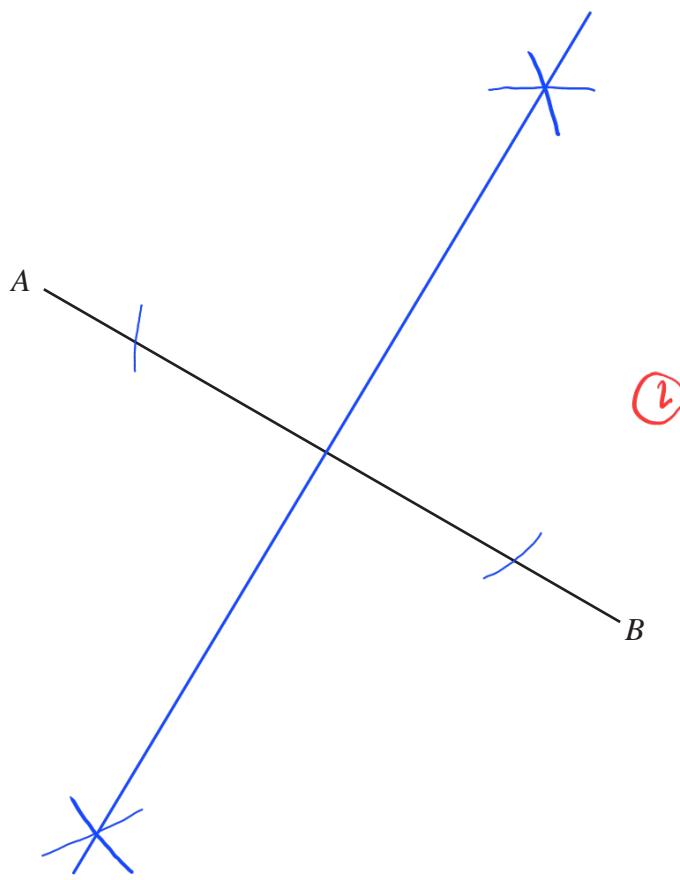
(Total for Question 6 is 2 marks)

Turn over ▶

7 Solve the simultaneous equations

$$3x + 5y = 6$$
$$7x - 5y = -11 \quad -①$$

$$x = \frac{6-5y}{3} \quad -②$$

Show clear algebraic working.

substitute ② into ① :

$$7\left(\frac{6-5y}{3}\right) - 5y = -11$$

$$7(6 - 5y) - 15y = -33$$

$$42 - 35y - 15y = -33$$

$$-50y = -33 - 42 \quad \Big) -42$$

$$\div 50 \left(\begin{array}{l} -50y = -75 \quad ① \\ \\ y = \frac{-75}{-50} = 1.5 \quad ① \end{array} \right.$$

$$x = \frac{6 - 5(1.5)}{3}$$

$$= -0.5 \quad ①$$

$x =$ .... -0.5

$y =$ 1.5

(Total for Question 7 is 3 marks)

8 Hamish buys a new car for $20 000
The car depreciates in value by 19% each year.

Work out the value of the car at the end of 3 years.
Give your answer to the nearest $.

Value of the car each year $= 100\% - 19\%$

$= 81\%$ (from the value at the start of each year)
①

Value of the car at the
end of year 3
$= 20\,000 \times \left(\frac{81}{100}\right)^3$ ①

$= 10\,629$ ①

$ .... $10\,629$

(Total for Question 8 is 3 marks)

8

**9**   The diagram shows a box in the shape of a cuboid.

Diagram **NOT**
accurately drawn

x cm

1.2 cm
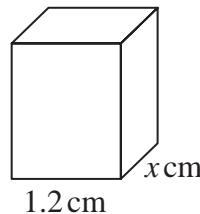
The box is put on a table.
The face of the box in contact with the table has length 1.2 metres and width x metres.

The force exerted by the box on the table is 27 newtons.
The pressure on the table due to the box is 30 newtons/m^2

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

Work out the value of x.

Area of the base of the box :

1.2 x m^2

30 N/m^2 = $\dfrac{27\ N}{1.2x\ m^2}$ ①

1.2x = $\dfrac{27}{30}$

1.2 x = 0.9

x = $\dfrac{0.9}{1.2}$ ①

= 0.75   ①

$x =$ ....0.75........

(Total for Question 9 is 3 marks)

P 5 9 8 1 7 A 0 9 2 8

10 The table shows information about the surface area of each of the world's oceans.

Ocean	Surface area in square kilometres
Pacific	1.56×10^8
Indian	6.86×10^7
Southern	2.03×10^7
Arctic	1.41×10^7
Atlantic	1.06×10^8

(a) Work out the difference, in square kilometres, between the surface area of the Atlantic Ocean and the surface area of the Indian Ocean.
Give your answer in standard form.

$\text{Atlantic} : 1.06 \times 10^8 = 10.6 \times 10^7$ ①

$\text{Indian} : 6.86 \times 10^7$

$10.6 \times 10^7 - 6.86 \times 10^7$

$= (10.6 - 6.86) \times 10^7$

$= 3.74 \times 10^7$ ①

................3.74×10^7................ square kilometres

(2)

The surface area of the Pacific Ocean is *k* times the surface area of the Arctic Ocean.

(b) Work out the value of *k*.
Give your answer correct to the nearest whole number.

$\text{Pacific} : 1.56 \times 10^8 = 15.6 \times 10^7$

$\text{Arctic} : 1.41 \times 10^7$

$\text{Pacific} = k \times \text{Arctic}$

$k = \dfrac{\text{Pacific}}{\text{Arctic}} = \dfrac{15.6 \times 10^7}{1.41 \times 10^7}$

$= 11$ ①

$k = $ ....11................

(1)

(Total for Question 10 is 3 marks)

11 (a) Write down the integer values of x that satisfy the inequality $-2 < x \leqslant 4$

$-1, 0, 1, 2, 3, 4$ ②

(2)

The region **R**, shown shaded in the diagram, is bounded by three straight lines.
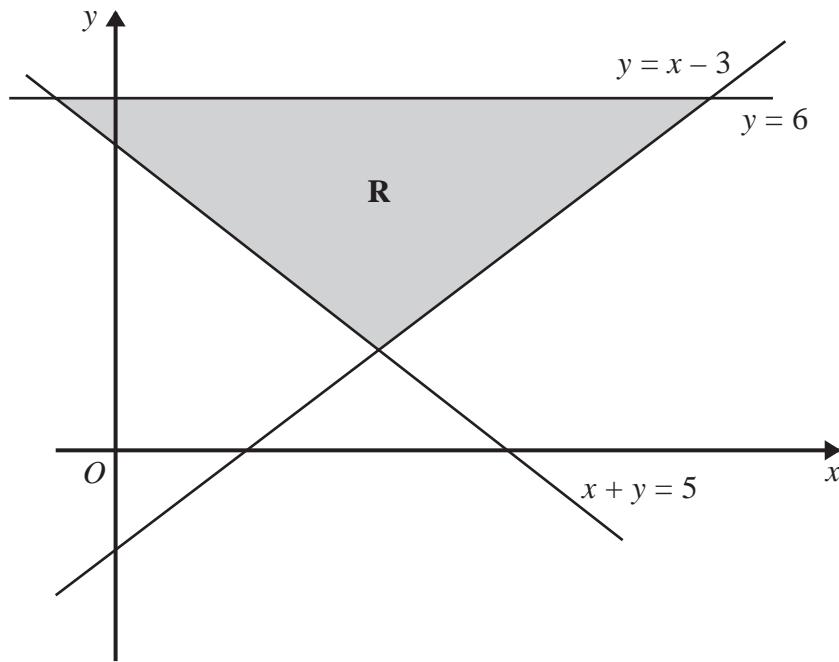

Diagram **NOT** accurately drawn

(b) Write down the three inequalities that define the region **R**.

$y \leq 6$

$x + y \geqslant 5$ ②

$y \geqslant x - 3$

(2)

(Total for Question 11 is 4 marks)

**12** The diagram shows two congruent isosceles triangles and parts of two congruent regular polygons, **X** and **Y**.

Diagram **NOT** accurately drawn

Polygon **X**

A $66°$

B $48°$

$66°$

C

Polygon **Y**

D $66°$

E

$66°$

F
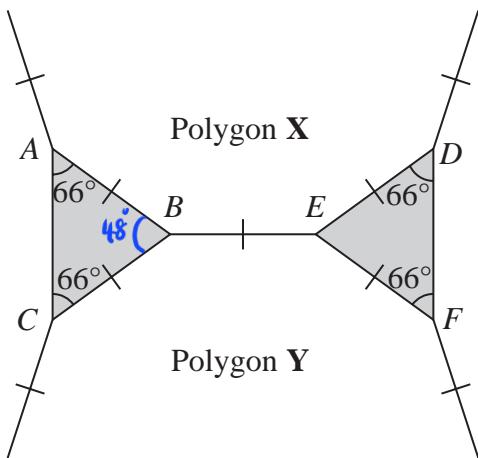
The two regular polygons each have n sides.

Work out the value of n.

angle ABC = 180° − 66° − 66°

        = 48° ①

Half of angle ABC = exterior angle of polygon X and Y

        = $\frac{1}{2} \times 48° = 24°$

Exterior angle of polygon = $\dfrac{360°}{\text{no. of sides}}$

$24° = \dfrac{360°}{n}$

$n = \dfrac{360°}{24°}$ ①

    = 15 ①

$n =$ ....15............

(Total for Question 12 is 3 marks)

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13
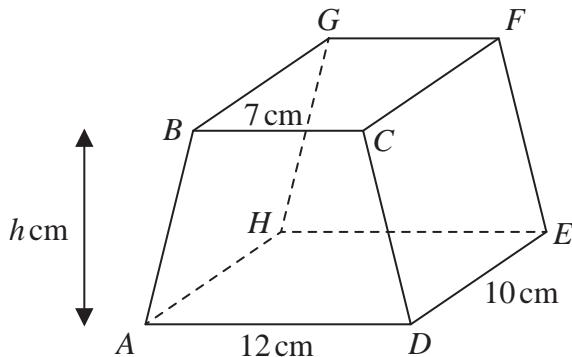
Diagram **NOT** accurately drawn

The diagram shows a prism *ABCDEFGH* in which *ABCD* is a trapezium with *BC* parallel to *AD* and *CDEF* is a rectangle.

$BC = 7$ cm $AD = 12$ cm $DE = 10$ cm

The height of trapezium *ABCD* is h cm
The volume of the prism is 608 cm^3

Work out the value of h.

Volume = area of trapezium × width

$\quad\quad = \frac{1}{2} \times (BC + AD) \times h \times DE$

$\text{Volume} = \frac{1}{2} \times (7 + 12) \times h \times 10 = 608$ ①

$\quad\quad\quad = 95h = 608$

$\quad\quad\quad\quad h = \frac{608}{95}$ ①

$\quad\quad\quad\quad\quad = 6.4$ ①

$h = $ .... 6.4 ....

(Total for Question 13 is 3 marks)

Turn over ▶

14 Max kept a record of the marks he scored in each of the 11 spelling tests he took one term.

Here are his marks.

18   5   7   12   11   18   15   16   17   13   14

Find the interquartile range of the marks.

Arrange the scores in order:

lower quartile   upper quartile

5 , 7, ⑪, 12, 13 , ⑭, 15 , 16, ⑰, 18, 18   ①

Q₁   median   Q₃
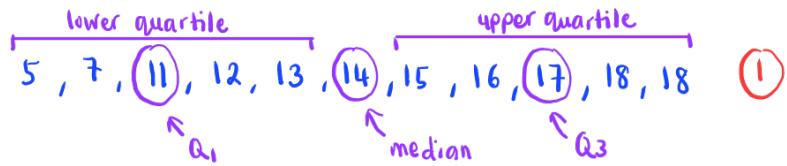
Interquartile range : Q₃ - Q₁

= 17 - 11   ①

= 6   ①

6

(Total for Question 14 is 3 marks)

P 5 9 8 1 7 A 0 1 4 2 8

15 (a) Complete the table of values for $y = x^2 - \dfrac{x}{2} - 3$

x	–3	–2	–1	0	1	2	3
y	7.5	2	–1.5	–3	–2.5	0	4.5

(2)

(2)

(b) On the grid, draw the graph of $y = x^2 - \dfrac{x}{2} - 3$ for values of x from –3 to 3
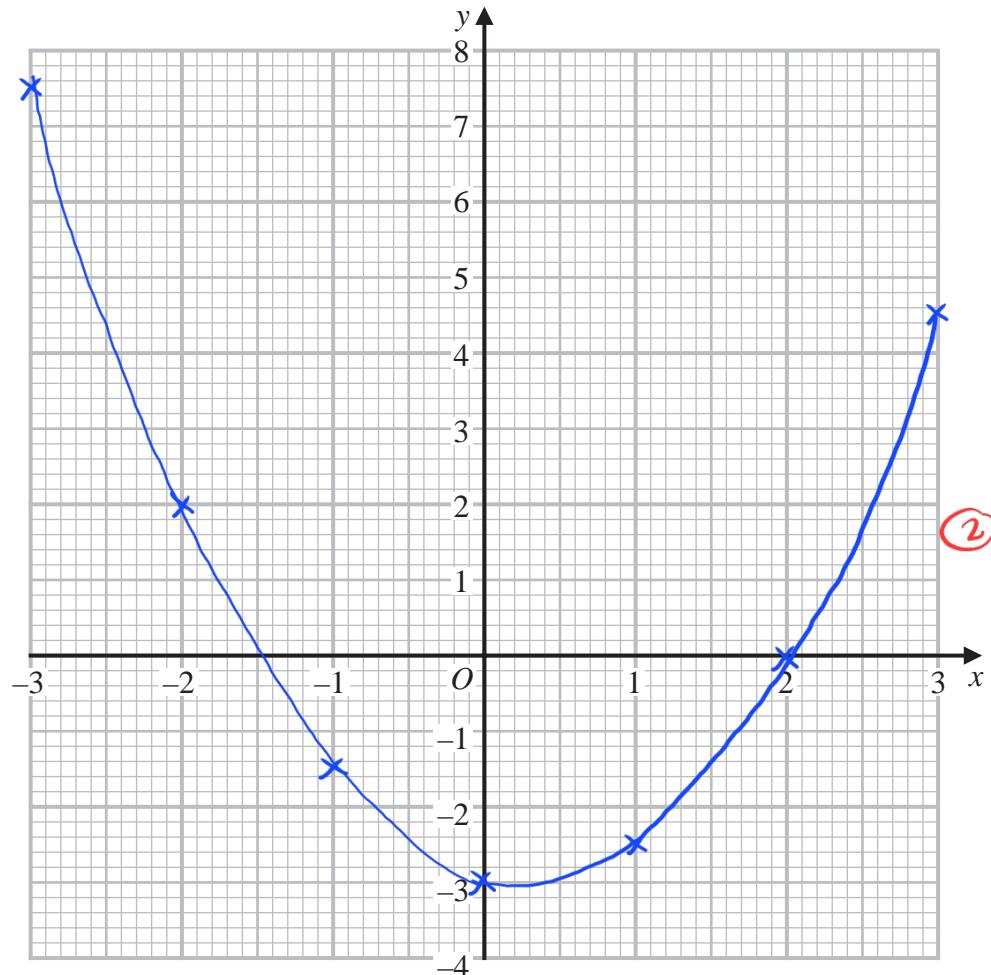
(2)

(2)

(Total for Question 15 is 4 marks)

16 Cody has two bags of counters, bag **A** and bag **B**.

Each of the counters has either an odd number or an even number written on it.

There are 10 counters in bag **A** and 7 of these counters have an **odd** number written on them.
There are 12 counters in bag **B** and 7 of these counters have an **odd** number written on them.

Cody is going to take at random a counter from bag **A** and a counter from bag **B**.

(a) Complete the probability tree diagram.
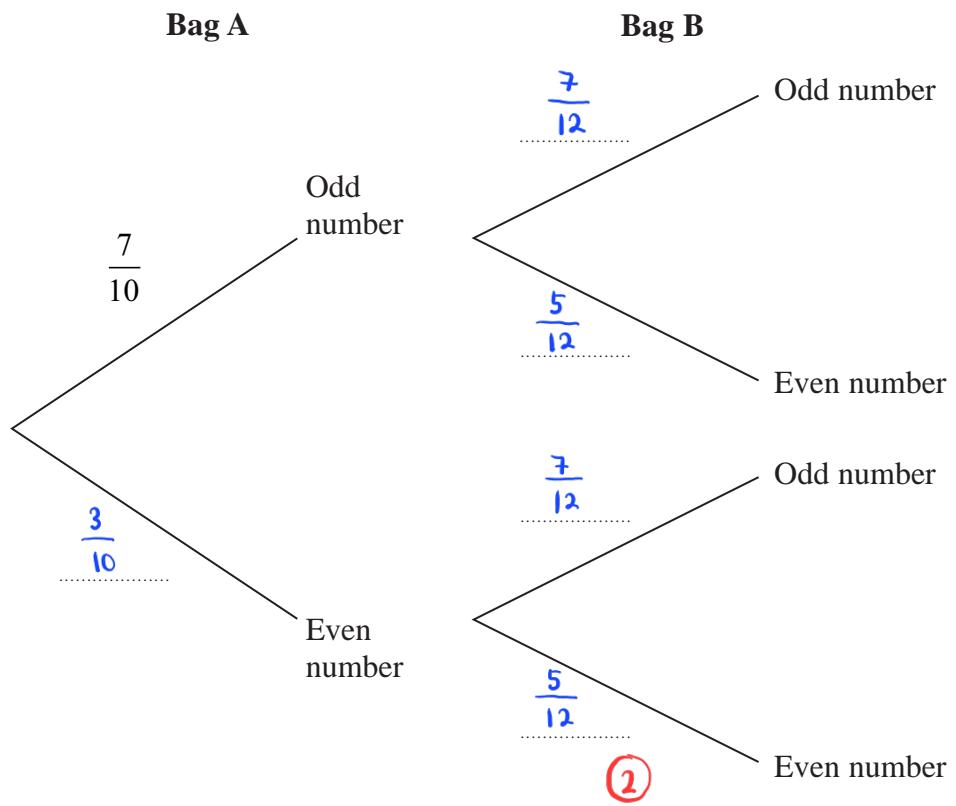
(2)

(b) Calculate the probability that the total of the numbers on the two counters will be an odd number.

To get a total of odd numbers,

① odd + even

② even + odd

① $\frac{7}{10} \times \frac{5}{12} = \frac{7}{24}$ ①

② $\frac{3}{10} \times \frac{7}{12} = \frac{7}{40}$

Total $= \frac{7}{24} + \frac{7}{40}$ ①

$= \frac{56}{120}$ ①

$\frac{56}{120}$

(3)

Harriet also has a bag of counters.
Each of her counters also has either an odd number or an even number written on it.

Harriet is going to take at random a counter from her bag of counters.

The probability that the number on each of Cody's two counters **and** the number on Harriet's counter will all be even is $\frac{3}{100}$

(c) Find the least number of counters that Harriet has in her bag.
Show your working clearly.

Let Harriet's even counter = E
Let Harriet's odd counter = D

$P(\text{all even}) = \frac{3}{10} \times \frac{5}{12} \times \frac{E}{E+D} = \frac{3}{100}$ ①

$\frac{E}{E+D} = \frac{0.03}{0.125}$

$\frac{E}{E+D} = 0.24$ ①

$E = 0.24E + 0.24D$

$0.76E = 0.24D$

$\rightarrow 6E$ and $19D$

$E = \frac{6}{19}D$

25

∴ Least number of counter is $6 + 19 = 25$ ①

(3)

**(Total for Question 16 is 8 marks)**

17 Some students in a school were asked the following question.

"Do you have a dog (*D*), a cat (*C*) or a rabbit (*R*)?"

Of these students

28 have a dog
18 have a cat
20 have a rabbit
8 have both a cat and a rabbit
9 have both a dog and a rabbit
x have both a dog and a cat
6 have a dog, a cat and a rabbit
5 have not got a dog or a cat or a rabbit

(a) Using this information, complete the Venn diagram to show the number of students in each appropriate subset.
Give the numbers in terms of *x* where necessary.

$\mathscr{E}$

5

D   *C*

$25-x$ $x-6$ $16-x$

6

3   2

③

9

R
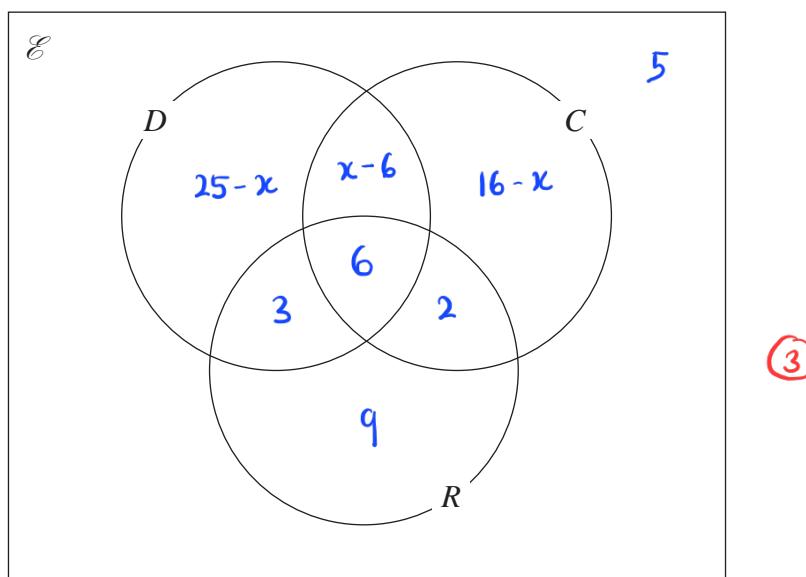
(3)

Given that a total of 50 students answered the question,

(b) work out the value of *x*.

$(25-x)+(x-6)+(16-x)+3+6+9+2+5 = 50$ ①

$60-x = 50$

$x = 60-50$

$= 10$ ①

$x = $ .... 10 ....

(2)

(c) Find n$(C' \cap D')$

↗ not C
↘ not D

9 + 5 = 14 ①

14

(1)

(Total for Question 17 is 6 marks)

18
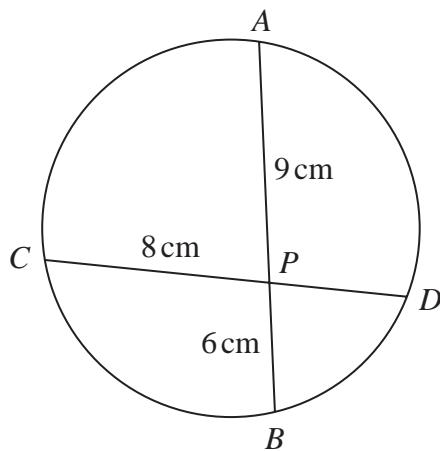
Diagram **NOT**
accurately drawn

APB and *CPD* are chords of a circle.

AP = 9 cm   *PB* = 6 cm   *CP* = 8 cm

Calculate the length of *PD*.

AP × PB = CP × PD

9 × 6 = 8 × PD ①

PD = $\dfrac{9 \times 6}{8}$

= 6.75 ①

6.75

.... cm

(Total for Question 18 is 2 marks)

19

Turn over ▶

19 (a) Solve $\dfrac{4-3x}{5} - \dfrac{3x-5}{2} = -3$

Show clear algebraic working.

$$2(4-3x) - 5(3x-5) = -3(5)(2) \quad ①$$

$$8 - 6x - 15x + 25 = -30 \quad ①$$

$$33 - 21x = -30$$

$$21x = 63$$

$$x = \frac{63}{21}$$

$$= 3 \quad ①$$

$x =$ .... 3 ....

(3)

(b) Solve the inequality $5y^2 - 17y \leqslant 40$

$$5y^2 - 17y - 40 \leqslant 0$$

$$(5y+8)(y-5) \leqslant 0 \quad ①$$

$$y = -\frac{8}{5} \quad \text{or} \quad y = 5 \quad ①$$

$$-\frac{8}{5} \leqslant y \leqslant 5 \quad ①$$
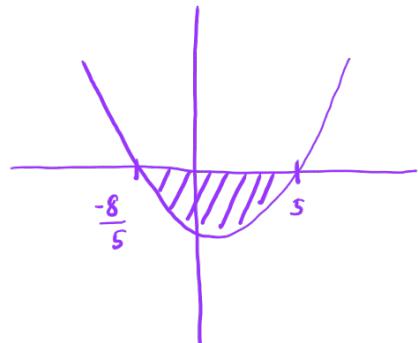
.... $-\frac{8}{5} \leqslant y \leqslant 5$ ....

(3)

(Total for Question 19 is 6 marks)

**20** The diagram shows two similar vases, **A** and **B**.
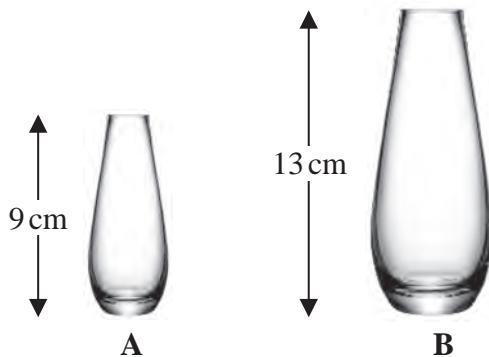
Diagram **NOT** accurately drawn

The height of vase **A** is 9 cm and the height of vase **B** is 13 cm.

Given that

$$\text{surface area of vase } \mathbf{A} + \text{surface area of vase } \mathbf{B} = 1800 \text{ cm}^2$$

calculate the surface area of vase **A**.

Comparing scale factor of A and B:

A : B

Height   9 : 13

Area 9^2 : 13^2

= 81 : 169

∴ $\dfrac{A}{B} = \dfrac{81}{169}$ ①

A + B = 1800

A + $\dfrac{169}{81}$ A = 1800   ①

$\dfrac{250}{181}$ A = 1800   ①

A = 583.2   ①

.......583.2....................... cm²

(Total for Question 20 is 4 marks)

21 (a) Simplify fully $\dfrac{10x^2 + 23x + 12}{4x^2 - 9}$

Factorising numerator :

$$10x^2 + 23x + 12 \equiv (5x+4)(2x+3) \quad ①$$

Factorising denominator :

$$4x^2 - 9 \equiv (2x+3)(2x-3) \quad ①$$

$$\frac{10x^2 + 23x + 12}{4x^2 - 9} \equiv \frac{(5x+4)(2x+3)}{(2x+3)(2x-3)}$$

$$= \frac{5x+4}{2x-3} \quad ①$$

$\dfrac{5x+4}{2x-3}$

(3)

$$2^{2y} \times 2^{3y+2} = \frac{8^{5y}}{4^n}$$

(b) Find an expression for n in terms of y.
Show clear algebraic working and simplify your expression.

$$8^{5y} = (2^3)^{5y}$$

$$= 2^{15y} \quad ①$$

$$4^n = (2^2)^n$$

$$= 2^{2n}$$

$$2^{2y} \times 2^{3y+2} = \frac{2^{15y}}{2^{2n}}$$

$$2^{2y+3y+2} = 2^{15y-2n} \quad ①$$

$$2y + 3y + 2 = 15y - 2n$$

$$5y + 2 = 15y - 2n \quad ①$$

$$2n = 15y - 5y - 2$$

$$2n = 10y - 2$$

$$n = 5y - 1 \quad ①$$

$n = 5y - 1$

(4)

(Total for Question 21 is 7 marks)

22 The first term of an arithmetic series S is –6
The sum of the first 30 terms of S is 2865

Find the 9th term of S.

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$T_n = a + (n-1)d$$

$$a = -6$$

$$S_{30} = 2865 = \frac{30}{2}\left[2(-6) + (30-1)d\right] \;①$$

$$2865 = 15\left(-12 + 29d\right)$$

$$191 = -12 + 29d$$

$$29d = 203$$

$$d = 7 \;①$$

$$T_9 = -6 + (9-1)7$$

$$= -6 + 8(7) \;①$$

$$= -6 + 56$$

$$= 50 \;①$$

50

....

(Total for Question 22 is 4 marks)

23 Express $7 - 12x - 2x^2$ in the form $a + b(x + c)^2$ where a, b and c are integers.

$$-2x^2 - 12x + 7$$

$$-2\left(x^2 + 6x - \frac{7}{2}\right) \;①$$

$$-2\left[(x+3)^2 - 9 - \frac{7}{2}\right] \;①$$

$$-2\left[(x+3)^2 - \frac{25}{2}\right]$$

$$-2(x+3)^2 + 25$$

$$\therefore 25 - 2(x+3)^2 \quad \text{where} \quad a = 25$$
$$\;① \qquad\qquad\qquad b = -2$$
$$c = 3$$

$25 - 2(x+3)^2$

....

(Total for Question 23 is 3 marks)

24 $\mathbf{L_1}$ and $\mathbf{L_2}$ are two straight lines.
The origin of the coordinate axes is O.

$\mathbf{L_1}$ has equation $5x + 10y = 8$
$\mathbf{L_2}$ is perpendicular to $\mathbf{L_1}$ and passes through the point with coordinates $(8, 6)$

$\mathbf{L_2}$ crosses the x-axis at the point A.
$\mathbf{L_2}$ intersects the straight line with equation $x = -3$ at the point B.

Find the area of triangle AOB.
Show your working clearly.

Equation of L_1 : $5x + 10y = 8$

$$10y = -5x + 8$$

$$y = -\frac{1}{2}x + \frac{4}{5} \quad \leftarrow \text{ in terms of } y = mx + c$$

Gradient of $L_1 = -\frac{1}{2}$

Gradient of $L_2 = 2 \quad \rightarrow \quad \boxed{m_{L_1} = \dfrac{-1}{m_{L_2}}}$ ①

Equation of L_2 : $6 = 2(8) + c$

$$c = -10$$

$$\therefore \quad y = 2x - 10 \quad ①$$

When L_2 crosses point A :

$$y = 0 \quad : \quad 0 = 2x - 10$$

$$x = 5 \quad ①$$

$\therefore L_2$ crosses point A at $(5,0)$

when L_2 intersects at point B :

$$x = -3 \quad : \quad y = 2(-3) - 10$$

$$y = -16$$

$\therefore L_2$ intersects at $(-3, -16)$
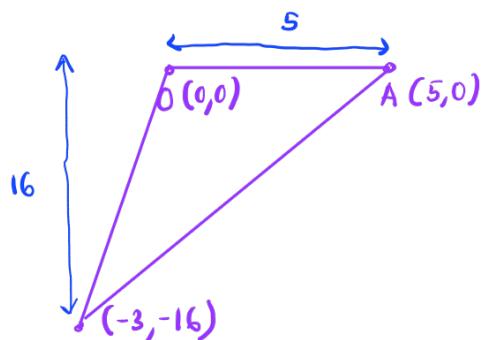
Area $= \dfrac{1}{2} \times 5 \times 16$ ①

$$= 40 \quad ①$$

40

(Total for Question 24 is 5 marks)

25 N is a multiple of 5

$A = N + 1$
$B = N - 1$

Prove, using algebra, that $A^2 - B^2$ is always a multiple of 20

$$A^2 = (N+1)(N+1)$$
$$= N^2 + 2N + 1 \quad \textcircled{1}$$

$$B^2 = (N-1)(N-1)$$
$$= N^2 - 2N + 1$$

$$A^2 - B^2 = (N^2 + 2N + 1) - (N^2 - 2N + 1)$$
$$= N^2 - N^2 + 2N + 2N + 1 - 1$$
$$= 4N \quad \textcircled{1}$$

since N is a multiple of 5,

4N is a multiple of 20. $\textcircled{1}$

$\therefore A^2 - B^2$ is always a multiple of 20.

(Total for Question 25 is 3 marks)

26 The diagram shows trapezium $OACB$.
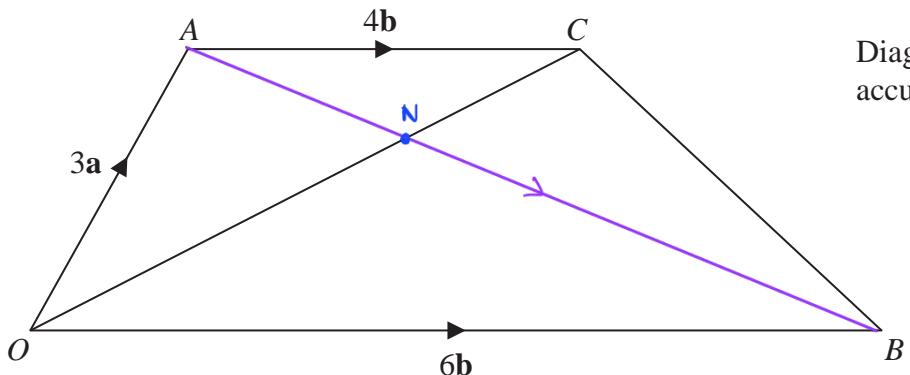
Diagram **NOT** accurately drawn

$\overrightarrow{OA} = 3\mathbf{a}$ $\qquad$ $\overrightarrow{OB} = 6\mathbf{b}$ $\qquad$ $\overrightarrow{AC} = 4\mathbf{b}$

N is the point on OC such that ANB is a straight line.

Find $\overrightarrow{ON}$ as a simplified expression in terms of $\mathbf{a}$ and $\mathbf{b}$.

$$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$$
$$= 3\underline{a} + 4\underline{b} \quad ①$$

$$\overrightarrow{ON} = x(\overrightarrow{OC})$$
$$= x(3\underline{a} + 4\underline{b}) \quad ①$$

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$$
$$= -3\underline{a} + 6\underline{b}$$

$$\overrightarrow{AN} = y(\overrightarrow{AB})$$
$$= y(-3\underline{a} + 6\underline{b})$$

$$\overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AN}$$
$$= 3\underline{a} + y(-3\underline{a} + 6\underline{b})$$

$$\therefore \quad x(3\underline{a} + 4\underline{b}) = 3\underline{a} + y(-3\underline{a} + 6\underline{b}) \quad ①$$

$\underline{a}$ term : $3x = 3 - 3y$
$$x = 1 - y$$

$\underline{b}$ term : $4x = 6y$
$$4(1-y) = 6y$$
$$4 - 4y = 6y$$
$$4 = 10y$$
$$y = \frac{4}{10} = 0.4 \quad , \quad x = 0.6 \quad ①$$

$$\overrightarrow{ON} = 0.6(3\underline{a} + 4\underline{b})$$
$$= 1.8\underline{a} + 2.4\underline{b} \quad ①$$

$$\overrightarrow{ON} = 1.8\underline{a} + 2.4\underline{b}$$
....

(Total for Question 26 is 5 marks)

TOTAL FOR PAPER IS 100 MARKS

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